

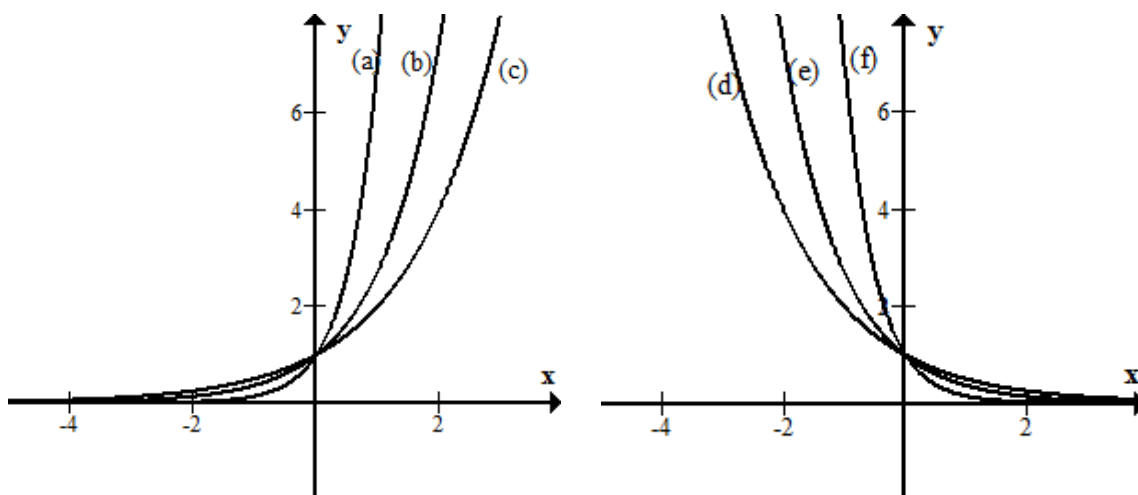
MAI

EXERCISES [MAI 2.5-2.6]
EXPONENTS AND LOGARITHMS - I
Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

EXPONENTIAL FUNCTIONS

1. [Maximum mark: 10]
 (a) The graphs of 6 functions are shown below



Match the graphs (a) (b) (c) (d) (e) and (f) to the following functions

$y = 2^x$	$y = 5^x$	$y = e^x$

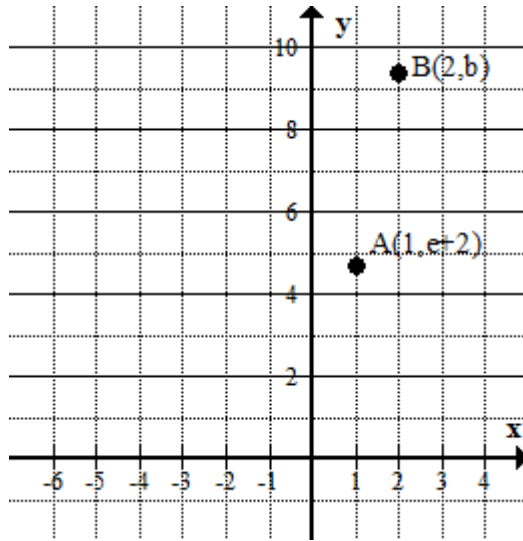
$y = 2^{-x}$	$y = 5^{-x}$	$y = e^{-x}$

- (b) Complete the following table which contains the common details for all six graphs

y – intercept	
Horizontal asymptote.	
Domain	
Range	

3. [Maximum mark: 6]

Let $f(x) = e^x + 2$. The points $A(1, e+2)$ and $B(2, b)$ of the graph are shown in the diagram below.



- (a) Write down the value of b . [1]
- (b) On the diagram above, sketch the graph of f . Indicate the y -intercept and the horizontal asymptote of the graph. [3]
- (c) Write down the domain and the range of f . [2]

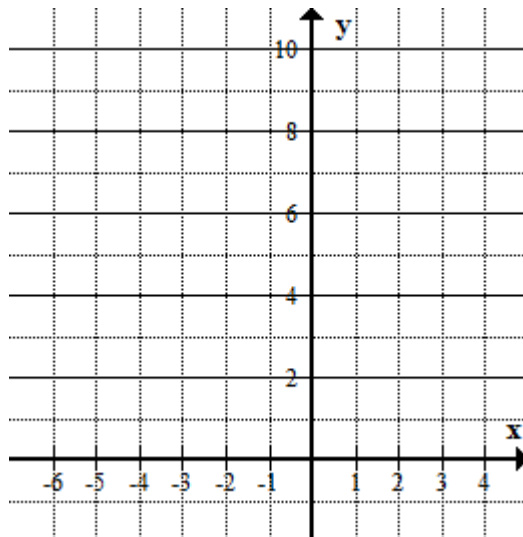
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4. [Maximum mark: 4]

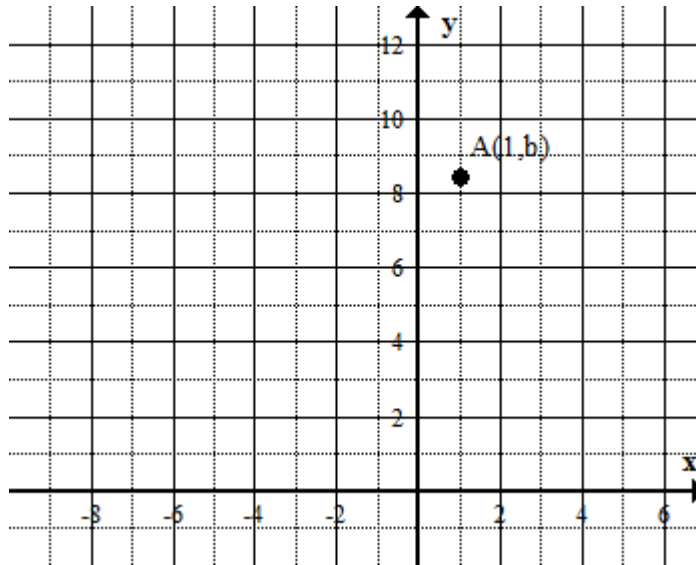
Sketch the graph of the function $f(x) = 2^x + 2$. Indicate the y -intercept, the horizontal asymptote and the points $A(1, a)$, $B(2, b)$ and $C(3, c)$ of the graph.



5. [Maximum mark: 12]

For each of the following functions sketch the graph and complete the table

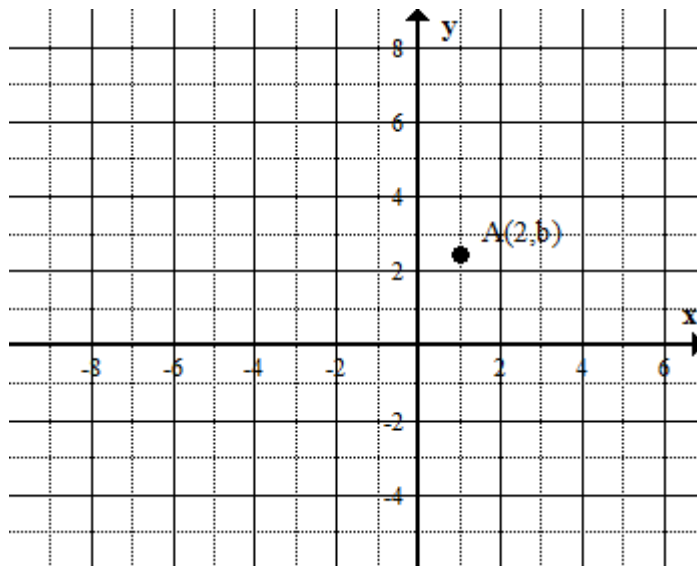
(a) $f(x) = 2e^x + 3$ (the point A lies on the curve).



y- intercept:	
horizontal asymptote:	value of b :
Domain:	Range:

[6]

(b) $f(x) = 2e^x - 3$ (the point A lies on the curve).



y- intercept:	x- intercept:
horizontal asymptote:	value of b :
Domain:	Range:

[6]

LOGARITHMS

6. [Maximum mark: 4]

(a) Write down the values of $x = \ln 3$ and $y = \log 3$ correct to 5 decimal places. [2]

(b) Find the values of e^x and 10^y to verify the results above. [2]

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7. [Maximum mark: 15] **[try to solve without GDC]**

Write down the following values

$\log 100 =$	$\log 10 =$	$\log 1 =$
$\log \frac{1}{100} =$	$\log \frac{1}{10} =$	$\log 0.1 =$
$\log 10^{2020} =$	$\log \sqrt{10} =$	$\log \sqrt[3]{10} =$
$\ln 1 =$	$\ln e =$	$\ln e^2 =$
$\ln \frac{1}{e} =$	$\ln \frac{1}{e^2} =$	$\ln \sqrt{e} =$

8. [Maximum mark: 6] **[try to solve without GDC]**

Find the value of x in the following equations

(a) $\log x = 3$ [2]

(b) $\ln x = 3$ [2]

(c) $\log(x + 1) = 3$ [2]

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9. [Maximum mark: 8]

By using your GDC find the value of the **integer** k in each of the following cases

(a) $\log 3k = \log 3 + \log 5$ (b) $\ln 3k = \ln 3 + \ln 5$ [2+1]

(c) $\log 21 = \log 3 + \log k$ (d) $\ln 21 = \ln 3 + \ln k$ [2+1]

Can you guess a formula for $\log xy$ and $\ln xy$? [2]

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10. [Maximum mark: 8]

By using your GDC find the value of the **integer** k in each of the following cases

(a) $\log 3^k = 5 \log 3$ (b) $\ln 3^k = 5 \ln 3$ [2+1]

(c) $\log 3^7 = k \log 3$ (d) $\ln 3^7 = k \ln 3$ [2+1]

Can you guess a formula for $\log x^n$ and $\ln x^n$? [2]

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11. [Maximum mark: 8] **[try to solve without GDC]**

Solve the equations

(a) $\log 2x = 2$ [2]

(b) $\log(2x + 4) = 1$ [3]

(c) $\ln(2x - 6) = 0$ [3]

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12. [Maximum mark: 6] **[try to solve without GDC]**

Let

$$a^x = b .$$

(i) If $A = \ln a$ and $B = \ln b$, then

$$Ax = B$$

(ii) If $A' = \log a$ and $B' = \log b$, then

$$A'x = B'$$

Given that $a = 2$ and $b = 8$

(a) Find the value of x [1]

(b) Verify the result (i) by completing the following [2]

$Ax =$	$B =$
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(c) Verify the result (ii) [3]

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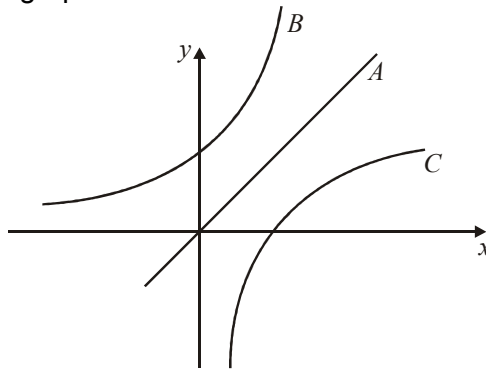
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13. [Maximum mark: 4]

The diagram shows three graphs.



A is part of the graph of $y = x$, B of the graph of $y = 2^x$, C is the reflection of graph B in line A . Write down:

- (a) the equation of C in the form $y = f(x)$. [2]
- (b) the coordinates of the point where C cuts the x -axis. [2]

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14. [Maximum mark: 6]

Let $f(x) = \ln(x + 2)$, $x > -2$ and $g(x) = e^{(x-4)}$, $x > 0$.

- (a) Write down the x -intercept of the graph of f . [1]
- (b) (i) Write down $f(-1.999)$. [3]
- (ii) Write down $g(4)$ [3]
- (c) Find the coordinates of the point of intersection of the graphs of f and g . [2]

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